A classic problem of probability

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The following text includes a very well known problem of probability theory.

Person P is the winner of a competition and is therefore allowed to compete for a car. The compere introduces P to three doors, labelled A, B, and C, respectively. Behind two of the doors, there are merely cheap toys to be found, but behind one of the doors, a grand new car is waiting for a new owner. Person P is to choose one of the doors, and will be given whatever is found behind it. P chooses door A. After that, the compere unexpectedly opens one of the remaining doors, B, hiding a toy. Then, Person P is given the choice to change to door C. In order to obtain as high probability as possible of winning the car, should Person P change to door C or stick to door A?

This problem is particularly interesting as its solution contradicts the logical intuition of many people. Instinctively, many people believe that the probability of winning the car, which originally equalled 1/3, does not change when the information about which one of the *other* doors is *not* hiding the car is presented. Because, indeed, the both remaining doors, the chosen and the remaining, did share the same *original* probability of hiding the car. However, most people would probably also agree on the fact that the probability would change, to 1/2, if the information were given before the initial door selection.

The problem with this intuitive “analysis”, however, is that one does not consider the possibility of *changing* door after the information has been presented. This, “new” discussion suggests that the probability indeed will equal 1/2, but this must be false as the original probability of *A’s* hiding the car was not 1/2.

To make the correct probability of winning the car after having changed the door more obvious, we will write out our premises and use those to deduce the probability.

1. Initially, the probability of *A’s* hiding the car was 1/3.
2. After the information about which one of the two remaining doors is *not* hiding the car has been presented, the following applies for a change of door:
    a. If the initial door did hide the car, then the new door will hide a toy.
    b. If the initial door hid a toy, then the new door will hide the car.

Thus, we realize that a change of door will make a loss out of a success and a success out of a loss. As the initial probability of winning the car was equal to 1/3 and all such cases will be converted to losses, the final (after a change of door) probability of loss will equal 1/3. Furthermore, if the probability of loss equals 1/3, then the probability of success must equal 2/3.

In order to obtain as high probability as possible of winning the car, thus, Person P should change to door *C*.