Institutionen för fysik, kemi och biologi

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A clear, concise, and rigorous treatment of classical physics and relativity theory?

Andreas Rejbrand

den 22 december 2011

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Sammanfattning
Abstract

This document is the formal report associated with a diploma work in theoretical physics. The aim of this diploma work is to write a book introducing classical physics (classical mechanics and electromagnetism) and relativity theory (both the special and general theories) in a 'clear, concise, rigorous, and coherent' manner. In this report, we will explain in detail the motivation for making such an attempt; in particular, we will define the standards that the book is supposed to fulfil, and we will give examples of how existing books fail to fulfil these. Then we will describe the process of writing the book, and, finally, we will compare the finished book against the standards that we have set forth.

Nyckelord
Keywords

theoretical physics, classical mechanics, electromagnetism, special relativity, differential geometry, general relativity

Abstract

This document is the formal report associated with a diploma work in theoretical physics. The aim of this diploma work is to write a book introducing classical physics (classical mechanics and electromagnetism) and relativity theory (both the special and general theories) in a 'clear, concise, rigorous, and coherent' manner. In this report, we will explain in detail the motivation for making such an attempt; in particular, we will define the standards that the book is supposed to fulfil, and we will give examples of how existing books fail to fulfil these. Then we will describe the process of writing the book, and, finally, we will compare the finished book against the standards that we have set forth.

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Table of Contents

1 Introduction

The aim of my diploma work is to write a text (a book) on classical physics (classical mechanics and electromagnetism) and relativity theory, which are my main areas of interest as far as physics is concerned. In this report, I begin with an exposition on the *motivation* for such an enterprise; this is the contents of Chapter 2. Among other things, I give some observations of the issues found in many existing physics textbooks. Naturally, one of the aims of my work is to overcome these issues. In Chapter 3, I describe the preparation I made before starting to work on my text. This chapter includes a list of the literature I consulted to learn differential geometry and relativity. In addition, I define the scope of my text, and the standards I wish the text to fulfil. In Chapter 4, I give details about the writing process and in Chapter 5 I compare the end result with the standards I set forth in Chapter 3. Finally, in Chapter 6, I discuss possible future work.

2 Motivation

In this chapter, I share my motivation for writing a book.

2.1 Some Issues found in Current Literature

Let us first observe some issues with the usual way students learn physics, which is also reflected in many common textbooks.

2.1.1 Lack of Coherency

Students learn physics in small steps. In particular, university-level studies in mathematics and physics are usually separated into distinct courses, each of which concerns only a single topic. An almost inevitable downside of this approach is that the presentation of mathematics and, perhaps even more so, physics becomes incoherent – different notations and approaches, perhaps mutually incompatible, are mixed. In addition, some basic ideas are introduced repeatedly, while some connections between seemingly different topics might end up untold, simply forgotten and 'lost' in the void between different courses.

2.1.2 Mathematics as the Language of Physics

The firm belief of the current author is that mathematics is the language of physics, and so one should learn mathematics *prior to* learning physics. This has a number of major advantages. First, if you know mathematics at a sufficiently advanced level, the formulation and analysis of physical systems will be tremendously simplified. Indeed, instead of deriving the mathematics required on the fly while treating a field of physics, you can simply refer to previous results, making the treatment shorter, sometimes *much* shorter. This is to a very high degree the case in quantum mechanics and general relativity, but the statement applies to essentially any topic of physics (and, more generally, any topic of exact science). This shortening will make the treatment of the physics easier to follow, and it will be easier to get an overview of the topic at hand.

Second, using this separation of mathematics and physics, one can easily see what is physics (physical postulates) and what is mathematics (deduction from postulates). This, of course, is crucial in understanding the theory. In addition, the overall process of learning physics is likely simplified, because the introduction of mathematical constructs is often easier to understand when they are introduced in a purely mathematical context. (At least this is how I feel, personally.) Again, the best example of this is probably in the fields of quantum mechanics and general relativity.

Third, since students normally don't study years of mathematics before they turn to physics, it is common that introductory courses (and textbooks) on physics are using a bare minimum of mathematical machinery. This may lead to results being presented in a less general and less elegant way. (An example is found below.)

Of course, university students *do* learn some mathematics, such as elementary calculus and linear algebra, very early in their studies, and so this knowledge is available for the majority of their physics courses. However, that is still not 'good enough' in my view. For one thing, the very first physics course is likely to be given at the very beginning of the studies, and so results like the formula for the displacement $x(t) = v_0 t + \frac{1}{2}at^2$ at time t caused by a constant acceleration a from an initial velocity v_0 might be deduced in some other way than the 'obvious' one (that is, by integrating $\ddot{x}(t) = a$ twice). If the student later on considers rotational motion, and learns that the *angular* displacement at time t is $\theta(t) = \omega_0 t + \frac{1}{2}\alpha t^2$ where ω_0 is the initial *angular* velocity and α is the constant *angular* acceleration, then he might (erroneously) think that the similarity in these formulae is due to some *physical* law, when the similarity is actually obvious from a purely *mathematical* point of view.

The issue just discussed is not a particularly big problem. What I am more concerned about is that the amount of mathematics studied prior to *advanced* physics courses is inadequate. From my own experience, I had some troubles learning quantum mechanics (abbreviated QM). I took the course already in my third year. Today when I think about it, I realise that I would have found the course far simpler if I had known more mathematics at that time. In particular, I would have appreciated

- **An honorary course in linear algebra**, discussing *complex* vector spaces,
- **complex analysis,**
- **Fourier analysis,** including elementary distribution theory,
- **abstract algebra,** especially group theory,
- **partial differential equations,** including special functions, and
- **functional analysis**, especially the theory of Hilbert spaces and special functions**.**

Had I taken these courses before I attended the course on quantum mechanics, that course would have been far easier, and I would have had much more mental resources left to appreciate the *physics* of the theory.[1]

I do realise that it would be 'unrealistic' to redesign the university-level physics programmes so that they start with 100 % mathematics the first two or three years, and then continue with 100 % physics for the remaining two or three years. That would probably scare away many prospective students that are interested in physics, but not so much in mathematics. *Nevertheless, this awareness does not stop me from writing a book aimed at those with a solid mathematical background who wish to see physics from the 'right' side!*

2.1.3 Lack of Rigor and Clarity

Although this might vary from person to person, to the current author, mathematical rigor not only makes arguments waterproof, it also makes them easier to understand. I don't want a quantity to have a nebulous definition or a statement to have several possible interpretations.[2]

[1] Some might argue that the course on quantum mechanics helped me appreciate the mathematical courses listed here. Although this is certainly true to *some* degree, I do insist that I would have preferred to study this mathematics before I took the course on QM.

2.2 A 'Perfect' Text

The main physical areas of interest of the current author are classical physics and the theory of relativity. These areas constitute a major part of the foundation of modern theoretical and applied physics (among the neglected parts are statistical physics and quantum mechanics, and a plethora of fields from less fundamental, and applied, physics), and I consider it fruitful to study them together. After all, both the special and the general theories of relativity extend Newtonian mechanics into new regimes. However, it is uncommon that textbooks treat such a wide range of physical theories. Indeed, almost without exception, the basics of classical mechanics are treated in a first physics course, while the topic of general relativity is rather left as an optional final-year course. This is one of my motivations of writing my book. Its sheer scope will make it almost unique, and, I hope, in a good way. Since I collect the theories in a single book, I can make the presentation *coherent*: I will use the same notation throughout the book (as far as possible), and I can always refer to results obtained previously, and make remarks about the connection between new and old results.

I will also separate the mathematics from the physics. As indicated above, this is a point I feel very strongly about. Purely mathematical ideas will be introduced in a natural, purely mathematical context. They will be studied carefully, in their own right, before we make use of them in a physical context. This will make the physical topics shorter, easier to follow (due to their shorter length and the increased amount of preparation), and it will be easier to be rigorous.

In short, I want to write the 'perfect' text on classical physics and relativity theory.

2.3 Survey of Existing Works

Perhaps the ideal text already exists? In this section, we will investigate some existing works and compare those to the ideals set forth above.

An excellent text on classical mechanics is (Taylor, 2005). This is a very clear and comprehensive treatment of the subject, and indeed, one of the current author's favourite physics books. However, it 'only' covers classical mechanics and special relativity, and not general relativity.

When it comes to general relativity (abbreviated 'GR'), the textbook used in the GR course I first took was (d'Inverno, 1992). In my view, this book has a number of disadvantages. First, it builds the introduction on special relativity (abbreviated 'SR') on the so-called 'k-calculus', which, I think, obscures the physics of the subject. I prefer instead a more classical approach to SR, focusing on the physical observations leading to the theory, and an (initially) naïve analysis of them. I think it is important to reflect the historical development of special relativity, since this includes the rationale for the theory. In particular, this includes the Maxwell theory of electromagnetism.

Second – and this is my strongest objection – d'Inverno introduces the concepts of tensors and manifolds in two chapters called 'Tensor Algebra' and 'Tensor Analysis'. I found these chapters practically useless. This is bad, because a high degree of confidence with manifolds and tensors is required in order to work with general relativity. At the time I participated in this class, my

[2] The worst example of a hard-to-read physics book that I know of is probably Kittel, *Introduction to Solid State Physics* (2005). I often have to guess what is meant, and perhaps I will find my working hypothesis to be invalid a few tens of pages later. The book has many good qualities, too, but the lack of clarity obscures the subject.

previous knowledge of differential geometry was almost entirely 'classical', and so I had very little experience with manifolds and tensors. Hence, I really *had* to learn these subjects. Eventually, I turned to other books on geometry, most notably (Frankel, 2004). But why was I so unsatisfied with d'Inverno's chapters?

In the beginning of the first of these chapters, d'Inverno writes, "We shall be concerned more with what you do with tensors rather than what tensors actually are". At that point, I got worried. How can one confidently work with quantities that one does not know what they are? Anyhow, I went on with the text. The chapter on tensor algebra is centred on the coordinate-based approach to tensors, and the definitions of the various kinds of tensors are motivated by a somewhat informal play with 'differentials' (as in 'infinitesimally small vectors and numbers'). Since I really wanted to learn and understand geometry, I was unsatisfied with both myself and with the book at the end of the first chapter.

The situation was certainly not improved when I reached the second chapter, 'Tensor Calculus'. In this chapter, two differential operations are defined, the Lie derivative and the covariant derivative. The definitions are motivated by rather informal, but non-geometrical, arguments (including plays with 'differentials'), and I found it hard to see the motivation for introducing these two concepts, and to see the differences and similarities between them. These definitions are followed by a number of pages containing technical calculations, but given the unsatisfying motivations for the two types of derivative operations, I found it hard to be motivated enough to read these pages. Indeed, why should I spend time doing computations with quantities I barely know what they are?

The rest of the book is reasonably clear, as soon as you have learned geometry from another source (such as Frankel), although it is not in the same class as Taylor's text. It should also be noted that d'Inverno does make it clear, in the introduction to the book, that he will use a less rigorous path in order to give the student a working knowledge of the (inherently non-trivial) theory of general relativity, and probably, to a large extent, he does succeed in doing this. Perhaps it would be more fair to say that the book didn't really work out well *for the current author*, rather than saying that the book itself is 'flawed'.

The (contemporary) advanced standard work on GR, I think, is (Wald, 1984). This book is more rigorous than d'Inverno (and hence, easier to understand), and so it does not suffer from the problems I had with d'Inverno. However, this is not the 'perfect' book I am looking for, since it contains (essentially) no classical physics, only very little SR, and, in addition, suffers from a slight deficiency of colourful illustrations (which happens to be a specialty of the current author).

2.4 Two More Incentives

Personally, I have been guided by two more incentives for writing my book. First, I feel that, to some extent, it is the *responsibility* of every generation of scientists to verify the theories obtained by previous generations of scientists. Second, and I have to admit that this probably has been my strongest motivation, I really want to *learn* geometry and relativity, and when it comes to learning a theory, there are few ways as effective as writing a text on the subject. (Another way that comes to mind is *teaching*.)

3 Preparation

In this chapter, I describe the preparation I made before writing the book. In particular, I give references to the books I read to familiarise myself with geometry and relativity, and I decide on the scope, the target audience, and the goals with my text.

3.1 Research

At the same time I took the class in general relativity, I read (Frankel, 2004); this 'bible' of geometrical physics introduces tensors as multilinear maps, and is generally more rigorous than d'Inverno's text. One of the most important benefits of Frankel is that it introduces the covariant derivative first in the context of surfaces embedded in $\mathbb{R}^3$, *where it has a very clear geometric interpretation*. (Finally I learnt what a covariant derivative actually 'is'!) I also read (Berry, 1976) at this time.

After the course, I went on with (Wald, 1984) and (Weinberg, 2008), two great books on general relativity and cosmology, respectively. I also read (Synge & Schild, 1949), but was not exceedingly impressed by the coordinate-based approach to tensors. To remedy this, I continued with (Conlon, 2001), a very rigorous text on differentiable manifolds. Then I fell in love with differential forms (or rather, I felt a need to learn these more carefully, especially since Frankel considers differential forms to be God's gift to vector calculus), and so I bought and read (Spivak, 1965), a very easy-to-read and concise text, followed by (Darling, 1994) and (Weintraub, 1997). The latter is kind of a 'differential forms for dummies', highly suitable as a complement to ordinary vector calculus, as indicated in the title of the book.

Eventually, I also bought and read (Ludvigsen, 1999), which is a very elegant text (at times, at least), with a decent introduction to tensors (as multilinear maps). While working on my book, I also consulted a few original papers, namely, (Einstein, 1905), (Hafele & Keating, 1972), (Hafele & Keating, 1972), and (Rossi & Hall, 1941). Finally, I read the article (Ellis & van Elst, 1998).

In total, I read almost 4 000 pages of geometry and relativity while preparing. A number of relevant other books are currently in my 'reading queue', most notably (Sattinger & Weaver, 1986) and (Hawking & Ellis, 1973). I also have bought (Fecko, 2006), but since I suspect this work to be rather similar in scope to Frankel, I have so far not felt much pressure to read it.

3.2 Choice of Topics

The topics I chose to cover in my book are

- classical mechanics,
- electromagnetism,
- special relativity,
- classical differential geometry of curves and surfaces in $\mathbb{R}^3$,

- modern differential geometry of manifolds,
- general relativity, and
- cosmology.

It is important to note that the list above reflects the *intended* chapters to be included in the text! As we will discuss in Chapter 5 ('The End Result'), I did not have time to write the last two chapters, the ones on general relativity and cosmology, before the deadline associated with the formal diploma work. Since this is the chapter about the preparation, the discussion below only reflects the *intended* list of chapters in the book.

Classical mechanics is 'obvious', since it lays the foundation for essentially all of physics. In addition, it is an extremely important (and fun!) subject in its own right, because more 'modern' theories, such as the special theory of relativity, the general theory of relativity, and quantum mechanics, only produces measurable deviations from classical mechanics in very extreme situations; the typical examples are at extreme speeds, extreme gravitational fields, and microscopic sizes, respectively.

Electromagnetism is also a part of classical physics, and of paramount practical importance. Indeed, electric and magnetic forces constitute a major part of our conception of the world. Maxwell's electromagnetism is a classical theory in the sense that it was developed early (in the 19$^{\text{th}}$ century) and it works well together with classical mechanics, unless you start scrutinizing the theory too much. However, somewhat ironically, the Maxwell theory of electromagnetism also forms the bridge to the special theory of relativity, since one can show that the Maxwell theory is *inconsistent* within a purely Newtonian theory of mechanics. In fact, if one postulates the Newtonian theory, it is possible to prove that Maxwell's equations *cannot* be valid. Thus, a chapter on classical electromagnetism naturally acts like a bridge to a chapter on the special theory of relativity.

The special theory of relativity supersedes the Newtonian theory at speeds that are not negligible compared to the speed of light, $c_0 = 299\,792\,458\ \mathrm{m/s}$. This theory is an experimentally very well-confirmed[3] theory, and it is used in (the making of) every-day consumer electronics.

The last part of the book is about the *general* theory of relativity, which supersedes the Newtonian theory in the presence of high gravitational fields, and on a cosmological scale. This theory is presented in the language of differential geometry, and therefore, I will consider this mathematical discipline before I turn to GR. Now, differential geometry comes in two forms, one classical theory of curves and surfaces in Euclidean $\mathbb{R}^2$ and $\mathbb{R}^3$, and one modern theory of tensor fields on general manifolds. I first give a rather complete introduction to the classical theory, which is easiest to understand, before I turn to the modern theory of manifolds and tensors. Then, after these two purely mathematical chapters, I turn to GR. Finally, equipped with the general theory of relativity, it is hard to resist writing a few pages on cosmology.

3.3 Choice of Target Audience

I will assume that the reader knows mathematics at the undergraduate level. In particular, the reader is assumed very well familiar with calculus (in one and in several variables), linear alge-

[3] Even in spite of the recent 'findings' at CERN, where neutrinos appear to be travelling marginally faster than light (Adam, T, et al. (OPERA Collaboration), 2011).

bra, and vector calculus. In addition, it is beneficial to know the basics of abstract algebra (particularly the concept of a 'group'), differential equations, Fourier analysis, functional analysis, and complex analysis. Differential geometry is treated in detail in the text, and so is not a prerequisite. The reader should however be familiar with basic classical physics; a first-term university course on classical mechanics and a similar course in electromagnetism should suffice. *Absolutely no prior knowledge of relativity theory (not even the special theory) is required.*

It might also be worth to point out that my text is likely to appeal to the mathematically inclined audience.

3.4 Choice of Standards

Below I list some of the foremost standards I have set forth for my work. You will recognise that most of them (except the last one) are simply the obvious solutions to the issues observed in Section 2.1.

- **Coherency.** Since all the topics listed in Section 3.2 are now treated under the same roof, an almost unique coherency is possible in the treatment.
- **Separation of Mathematics from Physics.** As obvious from the list in Section 3.2, I will treat classical differential geometry and modern differential geometry of manifolds in two separate chapters, which will be purely mathematical in nature.
- **Rigor and Clarity.** I will try to make the text as rigorous and clear as possible. Among other things, I will use a coordinate-free approach to tensors (considered as multilinear maps). I also hope that a general feeling of rigour will pervade the text.
- **Style and Clarity.** In my book, some propositions will have technical proofs. The vast majority of those are probably not required in order for the reader to follow the rest of the text. In addition, essentially every definition and proposition will be placed in its own box with a black, solid, border around it. My hope is that these two standards, too, increase the clarity. It should be easy to get an overview of a topic; you should be able to find the definitions of the quantities and their most important properties, without having to process a lot of technical details. I have also chosen to write physical postulates in such boxes with black, solid, borders. Indeed, the postulates of physical theories are very much like the axioms of mathematical theories, and so should be emphasised as much. The text also contains quite a number of examples, and these are enclosed in boxes not using solid border lines, but wavy lines.
- **Basics rather than Depth.** I am more interested in carefully treating the foundations of the theories than I am in proceeding very far with their applications. For example, I will spend much time discussing the basics of general relativity, but I will not find it particularly important to discuss all the aspects of black holes. This is also a point where my approach differs significantly from the one taken by d'Inverno, who wishes to cover as much as possible, including gravitational radiation, rather detailed analyses of black holes, and cosmology.

4 The Writing Process

Some details about the writing process follows.

4.1 Word-Processing Software

The book is written using Microsoft Word 2010, which is a really obnoxious[4] piece of software, due to a large number of restrictions and, as it seems, 'bugs'. Some annoyances are listed below.

- **Instability.** I make use of a very wide range of features, ranging from advanced paragraph formatting, cross-references, and automatic Table of Contents (TOC), index, and bibliography, to Word vector graphics, formulae, footnotes, and advanced page layout options. Microsoft Word crashed almost daily while writing the document, especially when I was editing formulae and Word vector graphics. In addition, formulae sometimes 'get corrupted', and you need to rewrite them from scratch to save the document.
- **Data Corruption.** Once a week or so, the document refuses to save; that is, suddenly the 'save' function does not work. At this time, the only choice is to discard the changes made since the last save, and revert to this old version. (Of course, one can copy the new paragraphs of plain text to the clipboard prior to doing this.) What is even worse is that, seemingly randomly, the saved files are corrupted, and cannot be opened the next time you try to open them. Typically, you find that the file size is remarkably low and so probably data is inevitably lost at these instances. To reduce the risk of data loss, I therefore saved the document once a minute while editing, and then I made a copy of the file to an external archive, retaining the previous version as well. As of now, this archive consists of 625 old versions of the text, summing up to 8.96 GB of data.
- **Sporadic Loss of Characters**. Sometimes single characters, especially in formulae, are missing when you open a previously saved document. Fortunately, this is rather uncommon.
- **Sporadic Changes of Formatting**. I want my vectors in **regular bold**; that is, I do not want them in *italics* or ***bold italics***. However, often when I open a previously saved version of the text, the most recently added vectors are suddenly in ***bold italics***, even though I am confident they were only **regular bold** when I saved the document the last time.
- **Visual Bugs**. It happens often that the text displayed on-screen does not match the position of the on-screen caret. This can be resolved by forcing a full redraw of the current view, e.g. by scrolling to a different page and then back, or by making a selection

[4] The views expressed in this section are solely the personal views of the current author, and do not reflect the official views of the Department of Mathematics (MAI), the Department of Physics, Chemistry and Biology (IFM), or the Linköping Institute of Technology (LiTH).

of the problematic paragraph(s). It also happens, if you use some particular paragraph settings, that some lines are rendered *twice* (erroneously).

- **Equation Numbering**. Microsoft Word 2010 has no feature to number equations. In fact, this was easier before the new formula editor was introduced in Microsoft Word 2007. Indeed, using the Equation Editor 3.0 OLE object in Office Word 2003 and earlier versions, you could create a centred tab stop at the middle of the page, and a right-aligned tab stop at the right margin, and then you could manually number your equations. This simple approach is not possible in Word 2007 and later, because if the formula is not alone on the paragraph, it will be shrunk to 'inline style'. Now, it seems that the only reasonable way to number an equation is to use a 3×1 table with a total width of 100 % of the page, and individual cell widths of 10 %, 80 %, and 10 % (say). The middle cell aligns its text at the centre, and here you put the formula. Since the formula is alone in its paragraph (indeed, in the entire cell), it will be rendered the right way. In the right-most cell, in which text is right-justified, you can manually write the equation number inside parenthesis. It should be noted that you can still use the Microsoft Equation 3.0 OLE object in Word 2007 and later, but since this equation editor is very restricted in its set of features, that is not an option. In any case, numbering equations manually in a 300+ page document is awkward, and, in addition, one often needs to make cross-references to equations. (I have similar issues with theorems and definition boxes.[5]) I need to develop some means to make this automatic, perhaps using VBA scripting.
- **Heading Numbering.** In Microsoft Word, it is possible to number headings. However, it is *not* possible to use different numbering formats in different parts of the document (such as sections). In particular, it is not possible to have a different numbering format for the appendices, which is a very natural thing to have. Of course, when I say that it is not 'possible', I mean that there is no automatic, or built-in, way of accomplishing this. There is a trick, however, as described in a Microsoft Office KB article.[6] Essentially, you reserve some particular heading for the appendices, such as Heading 7. (This means that you can only use Headings 1-6 as normal headings in the rest of the document.) Then you define that, in the document-wide heading-numbering scheme, the Heading 7 numbering should look different, e.g., instead of saying 'N.N.N.N.N.N.X' as usual, you let it say 'Appendix X', or 'A.X'. Naturally, you also change the format of Heading 7 so that it looks more like Heading 1 than the miniature heading that Heading 7 usually is. Now you can use Heading 7 as your main headings in the appendices. In the TOC, normally the seventh-order headings are not shown, and if they are, they will be attached to the last Heading 6 (or 5, or 4, or…), and so they will certainly not appear as the main ('Heading 1-style') headings that they are supposed to be. Fortunately, this can be solved by manually instructing the TOC field to render Heading 7 using the Header 1 format. You simply change

  { TOC \o "1-2" \h \z \u }

  (say) to

[5] http://superuser.com/questions/280410/using-fields-to-number-theorems-in-word-2010
[6] http://support.microsoft.com/kb/290953, retrieved 2011-12-10.

{ TOC \o "1-2" \h \z \u \t "Heading 7;1" }.

Needless to say, this 'trick' is far from an elegant solution. Some particular problems include the fact that you cannot have subheadings in the appendices, unless you tweak Heading 8 in an appropriate way. You can also tweak Heading 9 to obtain a third-order heading style in the appendices. But then you cannot create fourth-order headings, because there is no 'Heading 10'. In addition, there is no longer any automatic guarantee that the main-text Heading 1 (2, 3) has the same character format as the appendix Heading 7 (8, 9). Still, the trick does the job fairly well, and I am most gracious that Microsoft has written a KB article explaining the trick.

- **Problems with Cross-References.** Sometimes Word uses the term 'Figure' for figures, and sometimes it uses the Swedish term 'Figur' (since I use a Swedish version of Microsoft Office). This causes problems when you are to number and make cross-references to figures. Indeed, if Word at the moment uses the term 'Figure', you cannot make a reference to any figure, the caption of which you created while Word thought of an illustration as a 'Figur', and vice versa.

  Fortunately, this problem is easily fixed by means of a trivial VBA macro

```
Sub FixFigCaps()
    For Each Field In ActiveDocument.Fields
        If Field.Type = wdFieldSequence Then
            Field.Code.Text = Replace(Field.Code, "Figur ", "Figure ")
        End If
    Next Field
End Sub
```

  which will change every field

  { SEQ Figur * ARABIC }

  to

  { SEQ Figure * ARABIC }

- **Language in Fields.** It took me quite a while before I figured out how I could change the language of text automatically displayed in fields. For example, Word automatically can make references to particular pages in a book found in the (automatically generated) bibliography list of the document. But in my Swedish version of Word, the reference uses the term 'sida' instead of the English translation: 'page'. This is the case even if the language of the text at the caret is English at the time the reference is inserted, and even if the language of the 'Normal' style is English. Eventually, I figured out that the language can be changed manually by altering the field code. For instance, I can change

  { BIBLIOGRAPHY }

  to

  { BIBLIOGRAPHY \l 2057 }

to make the bibliography list use English terms (such as 'edition' instead of 'upplaga'). '2057' is the Microsoft Locale ID for English (United Kingdom) in decimal.[7]

- **PDF Export.** Some formatting features available in Word 2010 are lost when you export the document to a PDF file. One example is the triangle-wave border style.
- **Footnote + Continuous Section Break Bug.** I also encountered a bug that made a continuous section break cause a page break if there is a footnote earlier on the same page. This is actually a bug that is *confirmed* by Microsoft, and a partial workaround is given in a KB article.[8]

4.1.1 Bugs?

The first five points above are apparently bugs in the software. (The next last one can also be thought of as a bug, but you can also say something like "the PDF export is never guaranteed to create an identical document or support *every* feature of Word". Maybe you could even argue that the problem, for some reason, lies in the PDF format or the PDF viewer, such as Adobe Reader.)

The word 'apparently' is important, because the fact

(1) "I experience a lot of problems with the Microsoft Word 2010 program installed on my computer"

does not imply

(2) "The Microsoft Word 2010 software is buggy".

Indeed, it *might* be the case that I am the only one experiencing these problems. It is possible that there is some malfunctioning hardware (such as a RAM module) or some buggy driver installed on my computer, but I think that any such explanation is too far-fetched. For one thing, Microsoft Word is the *only* software that I have issues with, and although I do almost all my editing on my main computer (which is a high-end computer), occasionally I work with other computers as well, and I have seen similar instabilities in Word 2010 on these systems, too. I should also mention that I did use Word 2007 on my previous main computer, and already there I found Word to be buggy, especially when I was editing formulae. In particular, I remember that it was impossible to save a document after I had inserted a formula in a bulleted list containing a soft return.

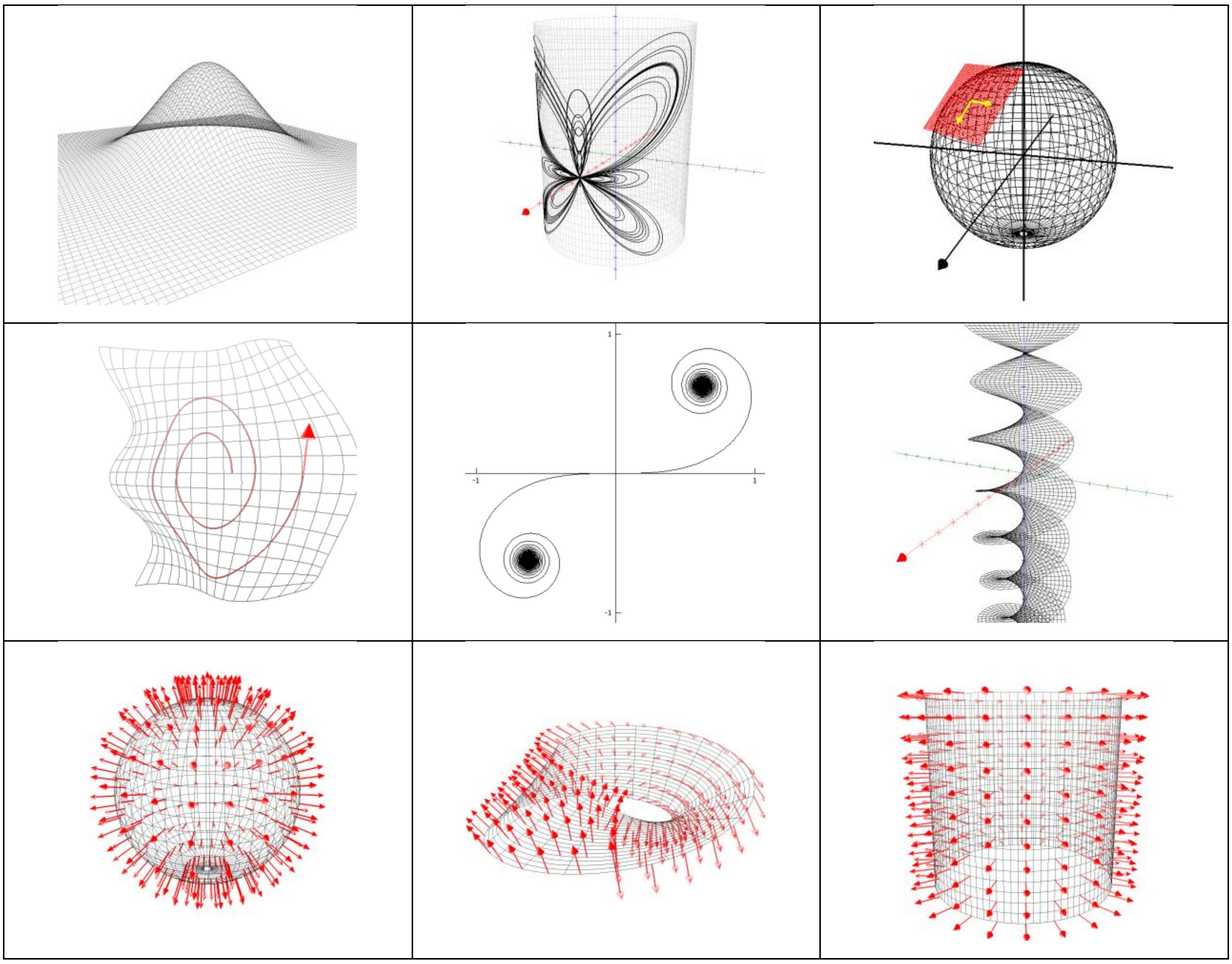
Therefore, my hypothesis, which I believe is a very strong one, is that Microsoft Word *is* buggy since the introduction of the new file format in Word 2007. The reason why such a heavily used product can be so buggy is probably that these bugs are invisible to the vast majority of users; indeed, most people do not write long, technical (mathematical) documents in Word.

Finally, let me clarify one thing: Why do I elaborate this much about issues in Microsoft Word? Well, if almost no one uses Word for long, technical documents, and if those who do do not complain about the issues, then they might never be fixed by Microsoft!

[7] Microsoft offers a list of their locale IDs at http://msdn.microsoft.com/en-us/goglobal/bb964664.aspx.
[8] http://support.microsoft.com/kb/292074

4.2 Graphics Software

The majority of all non-simple illustrations, including *all* three-dimensional computer-generated images, were created using my own software AlgoSim[9]. As indicated in Section 3.4, I have spent quite some effort on the graphics contained in the work. Below is a small selection of the images used (all of which are created in AlgoSim):



[9] www.algosim.se

5 The End Result

In this section, I shall discuss the end result, and to what extent my book fulfils the standards set forth in the previous sections.

5.1 The Meaning of 'End'

The phrase 'end result' is misleading, since I did not have time to finish the book before the formal deadline associated with the formal diploma work. My autumn (2011) has been very intense, with essentially no spare time, and I have been forced to make a number of sacrifices. Thus, recall that the phrase 'the end result' refers to the state of the book as of the formal deadline of the diploma work. The main part missing is the introduction to GR (but the two chapters on differential geometry are there). I intend to finish the book at a later time.

5.2 Statistics

The end result is a book consisting of 307 pages of A4 dimensions, and 83 000 words. The following table gives the distribution of text in different chapters. The discrepancy between the number of pages 307 (stated above) and 298 (stated in the table) is mainly due to the title page and TOC pages, which are not included in the table below.

Chapter	Number of Pages
Classical Mechanics	68
Classical Electromagnetism	20
Special Relativity	65
Classical Differential Geometry	78
Manifolds and Tensors	56
Appendices	11
Sum	**298**

5.3 The Title

I chose the title *Physics Done Right – An Attempt* for my text. The 'Physics' part needs no further explanation. Of course, as mentioned previously, I do not cover every single field of physics, but I do follow a thread from the very basics of physics (classical mechanics) to one of the modern generalisations, namely, general relativity. Therefore, I think, I can use a title as generic as this one. I could have chosen some other thread, such as classical mechanics to quantum mechanics, but, as noted in the introduction, my main interests are found in the former thread. This choice of 'thread' is not reflected in the title, but could be made apparent from the backside text, for instance.

The ‘Done Right’ part is inspired by the text (Treil, 2009) called *Linear Algebra Done Wrong*. I recall having read somewhere that Treil, in turn, chose his title as a pun on the title of the text *Linear Algebra Done Right* by Sheldon Axler, although I seem unable to find any reference where Treil confirms this.[10]

Finally, the ‘An Attempt’ suffix partly reflects the humbleness required by a good physicist. After all, there is no (known) ‘absolute truth’ in physics – a fact that has been repeatedly verified throughout history. Thus, any physicist claiming that he is doing everything right should not be taken too seriously. All one can do, in physics, is to make an attempt. However, the theories presented in my work are very well established (although I do present them in a slightly different way), and so the chief meaning of the ‘An Attempt’ part lies in the humbleness I feel as far as the *presentation* is concerned. I have been criticising d’Inverno’s text for not being sufficiently clear, but it should be noted that it is a non-trivial task to present the theory of differential geometry and general relativity in a clear yet concise way. I have *tried* to do so, but I wouldn’t dare to say that my text is ‘perfect’ in any respect. All I can say is that I have made *an attempt*.

5.4 Specific Features and Benefits

I wish to point out some particularly interesting approaches in, and particular strengths of, my text.

- **Classical Mechanics.** I have attempted to treat this subject in an as elegant and efficient way as possible; this means that I have *not* hesitated to use the required mathematical machinery. I also give attention to many of the ‘fun facts’ you find in nature, such as the fact that a hanging cable forms the graph of the hyperbolic cosine, while conic sections are found on a more astronomical level, but are still produced by the same fundamental force of nature, namely, gravity. I have also tried to give all the classical examples of simple mechanical systems, such as springs, pendula, and balls rolling down inclined hills. The high point of the chapter on classical mechanics is probably the derivation of Kepler’s laws of planetary motion. Another high point might be the use of the Gaussian formalism of classical gravity, from which many interesting (and non-trivial) results may be derived with almost no effort.
- **No Analytical Mechanics.** Since my goal has been to introduce the *basic physics* of our world, I have not made any use of, or even references to, analytical mechanics (such as Lagrangian and Hamiltonian mechanics). Indeed, as I see it, analytical mechanics is only a *formal reformulation* of Newtonian mechanics in order to simplify more complicated practical calculations. (There might be other benefits of analytical mechanics; for instance, analytical mechanics may provide bridges to areas of physics other than classical mechanics, but no such bridge would be within the scope of my text.)
- **Electromagnetism.** In the chapter on electromagnetism, all the classical laws (Coulomb’s law, the Biot–Savart law, and Faraday’s law of electromagnetic induction) are

[10] I do have a faint recollection of Treil writing about this point himself, but right now I am only able to find external references making this claim, e.g., http://www.math.uiuc.edu/~kapovich/416-11/416-11.html (accessed 2011-11-27 at 17.00).

discussed (although very briefly – recall that I do expect the reader to know the very basics of electromagnetism), as well as the Maxwell equations. I have then considered some simple geometries, and shown that the Maxwell equations are superior to naïve use of the classical laws in situations with symmetries (such as an infinitely long, charged cable). The proof of Maxwell's equations is trivial since we have already considered the step from Newton's law of gravitation to the Gaussian form of it in the preceding chapter on classical mechanics. The rest of the chapter is about electromagnetic radiation, and I emphasise the surprising discovery that light (most probably) is an electromagnetic wave. The chapter ends with the remarkable fact that the theory of electromagnetism is inconsistent when viewed from a Newtonian point of view.

- **Special Relativity**. I first introduce special relativity in the most naïve way imaginable (almost), and not until the end of the chapter, I reformulate the theory in a more concise language using four-vectors. This way I can emphasise the physical motivation for (and the consequences of) the theory before we develop a more efficient (but, perhaps, less transparent) machinery for it. I have made a major effort to make the basics of the theory completely clear. This is rather easy when it comes to SR kinematics (i.e., time dilation, length contraction, and the Lorentz transformation), but significantly more cumbersome when it comes to SR dynamics (and things like mass, momentum, and energy).

  In the section on SR dynamics, I show that the Newtonian law of momentum conservation *is incompatible with the Lorentz transformation*, and so *cannot* be valid when relativistic effects are taken into account. Then I define the relativistic momentum in the usual way (i.e., $\mathbf{p} = \gamma(u)m\mathbf{u}$), and, after a lengthy discussion on the concept of 'force' in SR, I define the relativistic kinetic, rest, and total energies of a particle (partly motivated by the discussion of forces). I argue that, even in Newtonian physics, the laws of momentum and energy conservation are merely postulates (indeed, the law of momentum conservation is essentially equivalent to Newton's third law), and so I postulate the conservation of relativistic energy and momentum. I finally show that the laws of conservation of relativistic momentum and energy *are* compatible with the Lorentz transformation. It also turns out that in order to prove the compatibility of either of these laws, the conservation of the other quantity in the initial frame has to be assumed. Thus, if we require momentum conservation, we need energy conservation, and vice versa. This is a rather important point, since the conservation of energy has the 'unexpected' consequence that the rest mass of particle is a measure of its internal energy, and so may change in ways that are highly unnatural from the point of view of Newtonian physics. In other words, the physical essence of the 'mass–energy equivalence' result $E = mc_0^2$ is a consequence of the aforementioned conservation laws. On the way to the above results, I discuss the two 'imaginable' definitions of the concept of 'force' in SR, I make detailed analyses of elastic versus inelastic collisions, and I notice that a man can travel to Vega in his lifespan. In essence, my main goal was to make the reader convinced that all the strange effects of SR are actually true effects: either they are logical consequences of previously postulated results, or they are experimentally verified, or, most commonly, they are a combination of these two. I end the SR chapter with an extensive treatment of four-vectors and the geometry of Minkowski space-time.

Accounts of SR dynamics as detailed as the one given in my text are far from omnipresent in introductory relativity texts. For example, d'Inverno treats SR dynamics in only nine pages, and starts by postulating that (1) the relativistic mass $m(u)$ of a particle is a function of its speed u, (2) this is 'the' mass of a particle, (3) the mass is conserved in collisions, and (4) the momentum $m(u)u$ is conserved, too. It then follows that $m(u) = \gamma(u)m_0$ where $m_0 = m(0)$ is the rest mass and $\gamma(u)$ is the Lorentz factor. There is essentially no discussion on compatibility of conservation laws, and the $E = mc_0^2$ result is obtained using hand waving (and the physical meaning of it is not explained very much), at least as far as I can see.

Taylor's account of SR dynamics is far more detailed, and contains (almost) everything found in my text. However, Taylor bases his approach on four-vectors, whereas I do all fundamental discoveries using ordinary three-vectors, which I hope is more transparent, and enhances the 'wow effect' felt when surprising consequences of the SR postulates are discovered. *Transparency* is the magic word in this case.

- **Special Relativity and Magnetism**. It is well-known that the theory of electric phenomena and the theory of magnetic phenomena can be combined into a single electromagnetic theory, indeed, Maxwell's theory. This is one way of thinking of the electromagnetic unification. However, using the special theory of relativity, it is possible to show that *it is impossible to have electric forces without any magnetic forces!* Thus, it is possible to show, not only that electric and magnetic phenomena can be described using a 'single' coherent mathematical theory, but that they really *are* the same thing, from a physical point of view. This observation seems to *require* the special theory of relativity.
- **Differential Geometry**. I have written *two* chapters on differential geometry. The first one is concerned with *classical* differential geometry of curves and surfaces embedded in Euclidean $\mathbb{R}^2$ or $\mathbb{R}^3$. The other chapter introduces tensors and manifolds, and reformulates the results of classical differential geometry in terms of this more 'modern' language. The idea, of course, is that the first chapter will be perceived as extremely simple and intuitive, while the latter chapter will 'translate' this intuition into the modern language of differentiable manifolds and tensor calculus. Perhaps the most important concept we will need from geometry is the covariant derivative, to be discussed next.
- **The Covariant Derivative.** The covariant derivative plays a central role in the modern treatment of differential geometry, and, in particular, in its main application (as far as we are concerned here), namely, general relativity. To make the introduction of this important concept as clear and painless as possible, I first introduce it in the special case of surfaces in $\mathbb{R}^3$, and, in addition, in the language of classical differential geometry, that is, without the notion of tensors, the summation convention, etc. This, I hope, will make this special case of the covariant derivative very easy to grasp, at least in principle. The term 'in principle' is important, for the explicit formula for the covariant derivative turns out to be rather long-winded when you do not make use of the summation convention. Thus, the choice of first introducing the covariant derivative in the classical language also has the benefit of serving as a motivation for the summation

convention. I also point out that the formula for the covariant derivative in terms of the Christoffel symbols has the advantage of not containing any explicit references to the embedding of the surface in the ambient 3-space, and this will be important when we later on will generalise the concept of the covariant derivative to general manifolds.

- **Verifying all the Details**. As a rule, I explicitly verify all the technical details that are often not considered in detail in other textbooks. For example, if u^a and v^a are two vector fields on a smooth manifold, then one can consider the commutator $[u,v]^a$ of those. It is clear that if u^a and v^a are vectors at each point on the manifold, then at least $[u,v]^a$ is a map taking functions to real numbers, again at each point of the manifold. But is it really a vector (i.e. a derivation) at each point, that is, is it linear and Leibniz? This is left as an exercise in Wald (c.f. Exercise 2.3a on page 27). Although, admittedly, many of these 'details' are trivial from a 'technical' point of view, they might be illustrative and useful for understanding the formalism. [I am *not* saying it is bad to put such details as exercises; I am only pointing out the fact that my text tends to include such details in the body matter.] This point is closely related to the 'Basics rather than Depth' choice above.
- **Colourful Illustrations.** I include a large number of colourful, computer-generated, 3D graphics in the book. These images were mainly created using my own software AlgoSim[9].

5.5 The Standards

Let us compare the ideals set forth in Section 3.4 above.

- **Coherency.** Since the entire text is written by the same author, and, in addition, at the same time (more or less), the coherency is good compared to the case where you combine different books from different authors. However, I did find it necessary to use one notation in the chapters on classical mechanics and geometry, and another in the chapters on modern geometry and relativity. Indeed, the classical theory deals with vectors $\mathbf{v}$ and scalars α, while the modern theory deals with arbitrary tensors. It simply is irresistible to use the familiar and very well-known classical notation in classical geometry and physics, and it is almost impossible to write a text about modern differential geometry without using tensorial notation (including the abstract index notation). I have, however, been very careful when I have introduced the tensorial notation. In particular, I made quite a discussion about the actual meaning of symbols like v^a and v^α and their context-sensitivity character. I also made an almost *too* big deal out of the summation convention, so that the reader will not forget it!

  Some specific examples of things gained by treating everything under the same roof are given below.

  - I start from the very basics, by introducing the concepts of space, time, and force, and state Newton's laws. I mention the fundamental forces. I then introduce Newton's law of universal gravitation and Coulomb's law, and the corresponding fields [from one of the particles], which I denote **G** and **E**, respectively. Then I consider the most common examples of kinematics (projectile motion, circular motion, the ideal spring, the simple pendulum, etc.). Hence, in all the remaining chapters, I can safely assume that the reader is perfectly well acquainted with these basics and, in addition, that all the notation has been settled.

- In the chapter on classical mechanics, I prove the Gaussian formula for gravity (i.e., $\oiint_S \mathbf{G} \cdot d\mathbf{A} = -4\pi GM$) starting from Newton's law of gravitation (i.e., $\mathbf{G} = -\frac{GM}{r^2}\hat{\mathbf{r}}$ in the 'field form' of it). This makes it extremely easy to deduce the Maxwell equation $\oiint_{\partial V} \boldsymbol{E} \cdot d\boldsymbol{A} = \frac{Q}{\epsilon_0}$ from Coulomb's law (that is, $\mathbf{E} := \frac{1}{4\pi\epsilon_0}\frac{Q}{r^2}\hat{\mathbf{r}}$ in its 'field form'), since the mathematics is exactly the same.
- In many[11] introductory relativity texts, the main motivation for SR, that is, the fact that Maxwell's equations are incompatible with the Galilean transformation (i.e., with Newtonian mechanics), is mentioned but not proven in detail (or, rather, 'at all'). In addition, in many introductory texts on electromagnetics, the relation to SR is not considered in detail. In my text, the connection *is* considered in detail, and this is very natural since both the Maxwell theory of electromagnetism and the special theory of relativity are well within the scope of the book.
- Since I introduce Newton's laws in the first chapter, and (will write about) GR in the last, I have plenty of opportunity to prepare for the GR chapter. Not only do I spend two entire chapters on differential geometry: Early on I also point out the curious fact that the force of gravity behaves differently compared to the other forces (such as the electrostatic force). Indeed, since the mass cancels in $ma = \frac{GMm}{r^2}$ [because the inertial mass equals the passive gravitational mass], the mass of a particle does *not* influence its trajectory in a gravitational field. On the contrary, the charge *and* mass determine its trajectory in an electromagnetic field.
- On the same note, I do believe that the two chapters on differential geometry, that are specifically written to simplify the introduction of GR, indeed *do* simplify it a lot. First I introduce the concept of curvature in the most intuitive way possible (that is, using classical differential geometry), and then I make a careful introduction to tensors and manifolds. Also, while discussing differential geometry, I do sneak in a number of GR-like calculations in 'disguise'.

- **Separation of Mathematics from Physics.** This I think I have managed to do very well. Indeed, the only mathematics I do *not* assume the reader is already familiar with is classical and modern differential geometry (almost), and these subjects are treated in separate chapters, which are purely mathematical in character. There is only one exception to the principle of separation, and that is when I derive properties of ellipses inside the section on planetary motion. The reason for this is simple: it would be 'silly' to write an entire chapter on the theory on ellipses, and in addition, the results about

[11] In fact, all I can think of, which includes d'Inverno, Taylor, Tipler.

ellipses that are required for Kepler's laws are not common knowledge. Fortunately, the topic of planar geometry is very familiar and far from intimidating, so I do not think that this exception to the principle will cause any serious problems.

Recall that I also assume the reader to be very well acquainted with calculus, linear algebra, and vector calculus, and hence I always try to introduce the physics in the most elegant, efficient, rigorous, general, and transparent way. Some examples:

- In the chapters on classical mechanics and electromagnetism, I use full vector notation from the very start, and I introduce the concept of *force fields* from the beginning.
- The work–kinetic energy theorem is proven in the perfect way (according to the adjectives listed above). Let me cite it here:

$$W \overset{\text{def}}{=} \int_\Gamma \mathbf{F}(\mathbf{x}) \cdot d\mathbf{l} = \int_0^1 \mathbf{F}\big(\mathbf{x}(t)\big) \cdot \dot{\mathbf{x}}(t)dt = \int_0^1 m\ddot{\mathbf{x}}(t) \cdot \dot{\mathbf{x}}(t)dt =$$
$$= m \int_0^1 \frac{d}{dt}\left(\frac{1}{2}\dot{\mathbf{x}}(t)^2\right) dt = \frac{1}{2}m\big(u_Q^2 - u_P^2\big) \overset{\text{def}}{=} \Delta E_k.$$

- I rely heavily on vector calculus when I discuss the motion of particles in conservative force fields, and when I introduce the Gaussian formalism of gravity (of course). I can do this very efficiently, since I assume the reader to know vector calculus already.
- The *first* example of kinematics concerns two massive bodies (masses m_1 and m_2) initially at rest relative to each other and separated by a distance d. I then ask how long it will take before they collide. The separation $r(t)$ between the bodies at time t is readily found to satisfy the ordinary differential equation $\ddot{r}(t) + kr(t)^{-2} = 0$ with initial conditions $r(0) = r_0 > 0$ and $\dot{r}(0) = 0$ [here $k = G(m_1 + m_2) > 0$]. This is a rather non-trivial non-linear ordinary differential equation, but the problem is well-defined and a solution can always be found numerically (given a precise value of r_0 and k), if one does not wish to solve it manually. The point is that I dare to choose the most interesting problems as examples, because I trust that the reader will not be intimidated by non-trivial mathematics. And, the point of the example is to solve a physical problem, not to demonstrate techniques for solving ODE's. Thus, the main part of the problem consists of *formulating* the initial-value (ODE) problem, not solving it. It is perfectly sensible to do that numerically. Still, I do solve it 'exactly' in an appendix. The solution $t \mapsto r(t)$ most likely cannot be expressed using elementary functions. Nevertheless, I do manage to reduce the ODE to a single ordinary equation (that is, a 'zeroth order ODE') for $r(t)$. I show that, in order to solve for $r(t)$, one has to invert the function

  $$p \mapsto \arctan p + \frac{p}{p^2 + 1}, \qquad p \geq 0.$$

  To proceed, I show that this function is injective and, thus, that there exists an inverse that I call $\mathcal{G}$, which is readily seen to be a map $\left[0, \frac{\pi}{2}\right[\to$

$[0, \infty[$. Then I can express $r(t)$ in terms of $\mathcal{G}$. Quite surprisingly, however, it turns out that the time at which the bodies collide [that is, the (smallest) time $t_{\text{collision}} > 0$ for which $r(t) \to 0$ as $t \to t_{\text{collision}}$] *can* be expressed solely in terms of elementary functions. In fact, I show that $t_{\text{collision}} = \frac{\pi}{2\sqrt{2k}} d^{3/2}$.

I also wrote a small Delphi[12] program that solves the ODE numerically[13], and a plot of $r(t)$ versus t is included in the main text (not the appendix) of the book.

- **Rigor and Clarity.** I think that I have succeeded fairly well to obtain clarity through rigor, but this is probably something for my readers to have a say about. I have tried to do all the details, and I do not hesitate before I use mathematical symbols[14] to make ideas precise. For instance, a curve $\gamma \subset \mathbb{R}^3$ is a subset of space and a corresponding f-curve is a function $\mathbf{r}: I \to \mathbb{R}^3$ such that $\gamma = \mathbf{r}(I)$ where $I \subset \mathbb{R}$ is some interval, just to mention one example of my usage of 'the language of mathematics'. This language makes it easier to be rigorous, and it is possible for me to use it because of the separation between mathematics and physics: I assume the reader already knows the symbols I use.
- **Style and Clarity.** The same remark applies here: the definite verdict should come from the readers. Still, the text contains a very large amount of boxes (definitions, theorems, postulates, observations, and examples). Notice in particular that I enclose important *observations* in boxes. I do not know really what to do with the triangle-wave borders, though. (Recall that these are destroyed when the document is converted to PDF.) Another 'invention' is that I use the symbol '$\coloneqq$' when I first define a quantity, and the symbol '$\stackrel{\text{def}}{=}$' when I wish to emphasise that equality holds by (a previous) definition. This is done to help the reader follow the arguments more easily. For example, if a formula reads $a \coloneqq$ (some expression), then he immediately knows that a is introduced at the present line, and so there is no need to go back and look for a previous definition of the quantity a. Later on, if he encounters the formula (some expression) $\stackrel{\text{def}}{=} a$, he knows that equality holds because of the way a has been defined; hence, there is no need to struggle to understand the equality.
- **Basics rather than Depth.** As mentioned above, I have tried to verify 'all' the details, and I have stuck to rather basic applications of the theories.

Finally, I can also mention that I have tried to keep a very high standard as far as the English language is concerned. I try to write *British* English consistently, I always make use of the 'Oxford comma', and I use logical punctuation.

"What are your favourite colours?" I asked her. "Red, green, and blue", she replied.

In addition, I always use *single* quotation marks, unless the text inside the quotation marks is an actual phrase that I am quoting. This usage is exemplified many times in this report.

[12] Delphi is an object-oriented programming language (and also the name of the associated compiler and IDE) based on Pascal and the compiler creates native Win32 applications.
[13] For $m_1 = m_2 = 2$ kg and $r_0 = 1$ m.
[14] I also consider English phrases like 'there exists' and 'for all/every' to be 'mathematical symbols'.

5.6 How to Obtain a Copy of the Book?

Until the book is finished, the most recent draft of it will be found at

- http://privat.rejbrand.se/PhysicsDoneRight.pdf.

When the book is finished, I will publish it on my public web site, www.rejbrand.se (and on the English version at http://english.rejbrand.se). More specifically, the book will be found on the 'Articles' page at

- http://rejbrand.se/rejbrand/documents.asp (Swedish version of the page) and
- http://english.rejbrand.se/rejbrand/documents.asp (English version of the page).

At this time, the file http://privat.rejbrand.se/PhysicsDoneRight.pdf will merely be a copy of the version at the public site.

6 Future Work

Let us start with the obvious: I need to finish the book. In particular, I need to

- write the chapter introducing GR,
- (possibly) write the chapter introducing cosmology,
- find a way to number theorems and equations,
- proof-reed the entire text, and
- consider how to publish it.

The last part is pretty simple, though: I'll probably just put it on my (big) website. Now that we have covered the obvious part of the 'future work' section, let's get more subtle.

There are two directions to go: 'left' or 'right'. By 'left', I mean that one could write books supposed to be read *prior* to this one, and by 'right', I mean that one could write books supposed to be read *after* this one. (Needless to say, I am thinking about how the books would be placed in a bookshelf.)

6.1 Left

The current author is tempted to write a book about the prerequisites to *Physics Done Right* (abbreviated PDR). It would be very enjoyable to write a book about elementary calculus (in one and several variables), linear algebra, and vector calculus. However, if I would undertake such an endeavour, I would *not* do it because of some perceived deficiency in the existing works in these fields; in fact, I know of many excellent books in these fields already. Rather, I would do it for 'fun' or to provide an alternative text (with a plethora of colourful illustrations, of course) on the subject(s).

6.2 Right

One can imagine several possible 'sequels', that is, books supposed to be read after *Physics Done Right*, and where you can assume the reader to know everything covered by PDR. Such enterprises are particularly interesting because of the 'Basics rather than Depth' principle used in PDR. For example, one could write about analytical mechanics (in the form of Lagrangian or Hamiltonian mechanics), which is a rather elegant theory, where you can almost automatically solve for the trajectories (in some configuration space) of complicated physical systems.

One could also make a sequel concerning general relativity. Indeed, there is *much* more to say about GR than is contained in (the intended finished version of) PDR, and then we are not talking about unbearably technical details or speculative theories, but important and fundamental, major, mainstream subjects. In *Physics Done Right*, we (will) stop soon after having formulated the very basics of GR and considered the very most typical examples.

6.3 Right Here

Is there anything I could do to improve *Physics Done Right*? As with any major work of art or science, there will always be room for improvements. I will probably want to do minor changes and tweaks in the text every now and then. In addition, since I am only human, I cannot guarantee that the text is free from misprints, or even mistakes that are even more serious. Nevertheless, at the present, I have two specific things in mind:

- I have never (really) learned the concept of an *affine space*. Still, when I wrote PDR, it did occur that I found it unnatural to introduce a distinguished point in space; in technical terms, I felt that the vector space might not be the ideal algebraic structure when it comes to modelling Newtonian space. Perhaps the concept of an affine space can be used to improve the presentation of the theories considered in the text.
- I have never (really) understood Noether's theorem. Perhaps it is possible to deduce stronger versions of the Newtonian law of energy conservation using it. What about the SR case? Anyhow (with or without Noether), I do think that my section on conservation of Newtonian energy needs some expansion to accommodate non-conservative forces, electromagnetic energy, and thermal energy.

6.4 Software

If I today would start writing on a new book in mathematics or physics, I would still choose Microsoft Word 2010. I am, and have always been, a keen Microsoft enthusiast, and my criticism of Word 2007 and 2010 is more of an exception than a rule when it comes to my attitude towards Microsoft products. I also assume that the observed issues in Microsoft Word will be solved in the next version of the software, or at least within a few years.

Moreover, I really enjoy working with AlgoSim, and will probably not change to any other mathematical software anytime soon. (Wolfram|Alpha[15] almost completely suits my CAS needs.)

[15] Wolfram|Alpha is Wolfram Research's free-to-use online CAS: www.wolframalpha.com.

A.1 Source Code

In this appendix, I include the Delphi source code for the numerical solver I wrote for the gravitational attraction initial-value ODE problem. Although the code is highly trivial, I include it for the sake of completeness. Syntax highlighting is done by my text editor Rejbrand Text Editor.[16]

The program utilizes the result

$$\ddot{r} + \frac{k}{r^2} = 0 \underset{\dot{r}<0}{\Longleftrightarrow} \dot{r} = -\sqrt{2k}\sqrt{\frac{1}{r} - \frac{1}{r_0}}$$

(derived in the appendix of the book) and integrates in steps in the most naïve way possible ($r \coloneqq r + \frac{dr}{dt} \cdot \Delta t$). Finally, the result is put on the clipboard in table form (rows separated by newlines (#13#10) and columns separated by horizontal tabs (#9)) suitable to be pasted into Microsoft Excel.

```
program GravInt;

{$APPTYPE CONSOLE}

uses
  SysUtils,
  Math,
  Clipbrd;

const
  G = 6.6726E-11;
  d = 1;
  m1 = 2;
  m2 = 2;
  k = G*(m1 + m2);

function RHS(r: real): real;
begin
  result := -sqrt(2*k)*sqrt(1/r - 1/d);
end;

const
  dt = 0.001;

var
  t: real;
  r: real;

  tend: real;
```

[16] http://english.rejbrand.se/rteditor

```
  i: integer;

type
  TRealPoint = record
    X, Y: real;
  end;

var
  vals: array of TRealPoint;
  NumItems: integer;
  S, S0: string;
  b: PAnsiChar;

const
  skip = 1000;

begin

  try

    tend := (pi/(2*sqrt(2*k))) * power(d, 3/2);
    SetLength(vals, round(tend / dt));

    r := d - 0.000001 (* why? *);

    NumItems := length(vals) div skip;

    Writeln('d      = ', d);
    Writeln('m1     = ', m1);
    Writeln('m2     = ', m2);
    Writeln('k      = ', k);
    Writeln;
    Writeln('tend   = ', tend);
    Writeln;
    Writeln('dt     = ', dt);
    Writeln('length = ', length(vals));
    Writeln('skip   = ', skip);
    Writeln('# itms = ', NumItems);

    Writeln;
    Write('Integrating... ');

    t := 0;

    for i := 0 to high(vals) do
    begin
      vals[i].X := t;
      vals[i].Y := r;

      r := r + RHS(r) * dt;
      t := t + dt;

```

```
      if r <= 0 then
      begin
        Write('[Hit r = 0 at i = ', i, '.] ');
        SetLength(vals, i + 1);
        break;
      end;
    end;
    Writeln('Done');

    Write('Formatting string... ');
    DecimalSeparator := ',';  // So that I can paste into Excel (sv-se)
    SetLength(S, 31 * NumItems);
    b := @S[1];
    for i := 0 to NumItems - 1 do
    begin
      S0 := FormatFloat('0.000000E+0000', vals[skip*i].X) + #9 +
        FormatFloat('0.000000E+0000', vals[skip*i].Y) + #13#10;
      Assert(length(S0) = 31);
      Move(S0[1], b, 31*sizeof(char));
      inc(b, 31*sizeof(char));
    end;
    Clipboard.AsText := S;
    Writeln('Done.');

    Readln;

  except
    on E: Exception do
    begin
      Writeln(E.Message);
      Readln;
    end;
  end;


end.
```

A.2 Bibliography

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